STABILITY OF CONTROLLED MOTION IN DIVING SIMULATIONS

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INTRODUCTION: Since the 1996 Atlanta Olympic games the regulations in some sports disciplines have been changed. The degrees of difficulty are not as limited as before. Coaches as well as athletes want support in creating new motions and need information about the feasibility and stability of these motions.
The present paper deals with the motion of an off spring board diver, its modeling, simulation and control using multibody system dynamics. The twisting somersault motion of a diver in free flight is simulated using a standard man model based on an anthropomorphic multibody system of rigid segments (MBS). The main goal is to give a Lyapunov-stable dynamic control law of a tree-like MBS having a degree of freedom n and moving along an m-dimensional submanifold of the n-dimensional configuration space ($0<m\leq n$). The prescribed reference motion defining that submanifold is obtained by kinematic analysis of video sequences of real motions. Consistent initial values for the free flight are evaluated by nonlinear optimization in order to minimize the deviations of the marker points from the measurement data.
The dynamic investigations in this paper are based on the Saziorski man model (see [1]) completed and used by the Institute of Mechatronics Chemnitz in several research projects of biomechanics, especially in sports, rehabilitation and accident mechanics (see [4]).

The Reference Motion: A special problem of motion control in sports is to define a reference motion. We define the reference motion by kinematic analysis of video sequences of real motions. Unfortunately, the use of inverse kinematics does not supply good results. The reason is that the measurement data of the time history of marker points fixed at the diver generally lead to wrong driving torques calculated by inverse dynamics (even after smoothing data). Therefore, a special approach developed at the Institute of Mechatronics is used to get better results, namely a sufficiently smooth motion and quite correct driving torques acting in the joints of the man model. That approach uses so-called dynamic tracking: the 3-dimensional man model is embedded in the set of marker points generated by video records. Force-coupling is used to approximate the marker points by corresponding model-fixed points. The numerical integration of Lagrange's equations of motion yields the desired smooth time history of the reference motion. The m-dimensional submanifold $\mathbf{V}^m$ of the n-dimensional configuration space $\mathbf{V}^n$ on which the man model has to move can be defined by rheonomic constraints

$$f^{a_1}(q,t) := q^{a_1} - q_0^{a_1}(t) = 0, \qquad \left|\{a_1\}\right| = r := n - m \tag{2.1}$$

where q^{a_1} denote the intrinsic generalized coordinates (relative coordinates of the joints of the man model) and $q_0^{a_1}(t)$ denote the time history of the reference motion obtained by dynamic tracking as described above.

A Lyapunov-stable Force Control Law: Now the main goal is to define a Lyapunov-stable control law to get the measured motion of the man model moving along the submanifold $\mathbf{V}^m$.

In a first step the calculated time history of the intrinsic coordinates $q_0^{a_1}(t)$ is used to simulate the free flying diver, i.e., the man model is partially kinematically controlled by prescribing the intrinsic coordinates q^{a_1} as functions of time. Then the remaining m=6 external coordinates q^{a_2} characterize the absolute position and orientation of the reference body pelvis. This motion is strongly influenced by the corresponding initial velocities: they define the total linear momentum as well as the total angular momentum of the man model. Therefore, in the case of kinematic control of the intrinsic coordinates, the initial velocities corresponding to the 6 external generalized coordinates q^{a_2} must be defined in such a way that the motion of the diver prescribed in the 3-dimensional Euclidean space is approximated in the best possible way. This can be done by using nonlinear optimization. The cost functional which must be minimized is defined by the maximum of the squared distances between video-generated marker points and corresponding body-fixed points of the man model over a certain time interval.

In the second step the motion of the diver should no longer be kinematically constrained, but instead controlled by forces and torques in the intrinsic joints of the man model. Usual inverse dynamics by applying the calculated generalized forces leads to unstable behavior of the motion after a short time. Therefore, a dynamic feedback control law based on the Voronetz-equations (Lagrange's equations projected onto the submanifold of a constrained mechanical system) is used.

We have generalized the so-called augmented PD-control approach known from nonlinear control theory (see [3]) to the case in which the nominal time history is known only for the intrinsically generalized coordinates q^{a_1} but not for the external coordinates q^{a_2}. Our method uses the fundamental differential-geometric concepts and methods of Lagrangian multibody dynamics (see [2]).

Let n be the degree of freedom of a given MBS. The representing point of the MBS is moving in the n-dimensional configuration space $\mathbf{R}^n$, which becomes a Riemannian space $\mathbf{V}^n$ by introducing a Riemannian metric g_{ab} and corresponding Christoffel symbols of the first kind Γ_{abc}. The motion equations are Lagrange's equations of the second kind in explicit form:

$$g_{ab}(q)\ddot{q}^b + \Gamma_{abc}(q)\dot{q}^b\dot{q}^c = Q_a(\dot{q}, q, t). \tag{3.1}$$

Q_a denote the generalized forces. Let the submanifold $\mathbf{V}^m$ (in case of the diver m=6) be defined by (2.1).

The task is to find a Lyapunov-stable position control law for tracking along $\mathbf{V}^m$, i.e., define a control force R_a such that q(t) $\in \mathbf{V}^m$ or (q(t), $\dot{q}(t)$) $\in T^m \mathbf{V}^m$, the tangential bundle corresponding to $\mathbf{V}^m$. To this end Lagrange's equations (3.1) are divided into two parts due to the partitioning of the generalized coordinates (q^a) = (q^{a_1}, q^{a_2}):

$$g_{a_1b_1}(q)\ddot{q}^{b_1} + g_{a_1b_2}(q)\ddot{q}^{b_2} + \Gamma_{a_1bc}(q)\dot{q}^b\dot{q}^c = Q_{a_1} + R_{a_1}, \tag{3.2a}$$

$$g_{a_2b_1}(q)\ddot{q}^{b_1} + g_{a_2b_2}(q)\ddot{q}^{b_2} + \Gamma_{a_2bc}(q)\dot{q}^b\dot{q}^c = Q_{a_2} + R_{a_2}. \tag{3.2b}$$

With respect to the special type of constraints (2.1), the augmented PD control law is given by:

$$R_{a_2} \equiv 0, \tag{3.3a}$$

$$\begin{aligned} R_{a_1} = {} & g_{a_1b_1}\ddot{q}_0^{b_1}(t) + g_{a_1b_2}g^{b_2a_2}\left(Q_{a_2} - g_{a_2b_1}\ddot{q}_0^{b_1}(t) - \Gamma_{a_2bc}\dot{q}^b\dot{q}^c\right) + \\ & + \Gamma_{a_1bc_1}\dot{q}^b\dot{q}_0^{c_1}(t) + \Gamma_{a_1bc_2}\dot{q}^b\dot{q}^{c_2} - Q_{a_1} - K_{a_1b_1}\dot{e}^{b_1} - C_{a_1b_1}e^{b_1}, \end{aligned} \tag{3.3b}$$

$e^{b_1} := q^{b_1} - q_0^{b_1}(t)$ denotes the error, $K_{a_1b_1}$ and $C_{a_1b_1}$ denote symmetrical and positive definite gain matrices characterizing the feedback component to reduce tracking errors.

The control law R_{a_1} is stable in the sense of Lyapunov, that means errors e^{b_1} and their derivatives $\dot{e}^{b_1}$ will tend to zero asymptotically. Figure 1 shows the result of this procedure for the case of the diver's free flight. The man model moves along the manifold defined by the reference trajectory $q_0^{a_1}(t)$, the nominal time history of the intrinsic joint variables of the model, the initial velocities being optimized in the above described way. The conservation of the total angular momentum with respect to the instantaneous center of mass of the man model in free flight is indicated in Figure 1. The time history of the generalized control forces R_{a_1} can be monitored, which yields the desired forces and moments in the joints of the model.

CONCLUSIONS: The twisting somersault motion of a diver in free flight is simulated using a standard man model based on an anthropomorphic MBS of rigid segments. The paper presents a Lyapunov-stable dynamic control law of a tree-like MBS having the degree of freedom n and moving along an m-dimensional submanifold ($0<m\leq n$) given by a reference motion. The approach essentially uses differential-geometric concepts and methods well-known from Lagrangian Multibody Dynamics. Applications are possible in other sports simulations, as for example, ice-skating and gymnastics.

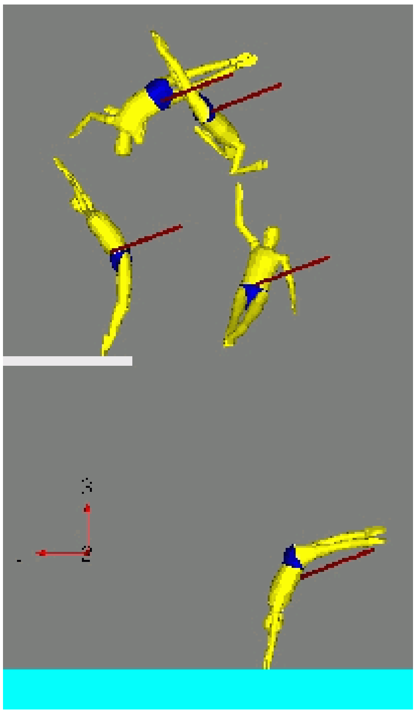

Figure 1: Diver in free flight

This research project was partially supported by the Federal Institute of Sport Science, Cologne, Germany.

REFERENCES:

[1] Aruin, A. S., Saziorsky, W. M. (1989). Ergonomic Biomechanics. Moskwa: Mechanical Engineering.
[2] Maisser, P. (1997). Differential-Geometric Methods in Multibody Dynamics, Nonlinear Analysis. *Theory, Methods & Applications* **30**(8), 5127–5133.
[3] Murray, R. M., Li, X., Sastry, S. S. (1994). A Mathematical Introduction to Robotic Manipulation. Boca Raton, Ann Arbor, London, Tokyo: CRC Press.
[4] Wolf, C.-D., Parsche, U., Bikowski, S. (1998). Der Mensch als mechanisches Mehrkörpermodell in Alaska. IfM-Report, Institute of Mechatronics at the Technical University Chemnitz-Zwickau.