SYSTEMATIC ERRORS IN DETERMINING THE CENTER OF PRESSURE (COP) WITH FORCE PLATES DEPENDING ON THE LOAD DISTRIBUTION

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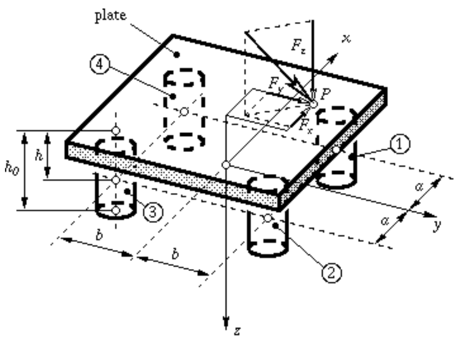

Figure 1: *Schematic diagram of a force plate. (1…4) measurement posts with tri-axial force transducers at coordinates [±a, ±b, 0]^T; h_0 total height of the force plate; h distance of the surface of the plate from the plane of measurement; P point of force application of a point load at coordinates [x_P, y_P, z_P] with z_P=h.*

INTRODUCTION: Systematic errors of up to 20mm and more have been reported in determining the center of pressure (COP) with force plates [1]. Experimentally these errors are determined by applying a point load to the force plate at a defined point and comparing the true coordinates of this point with the coordinates calculated from the signals of the force plate. The correction formulas given in the literature use polynomial approximations based on these measurements [4]. From analytical models it is known that the errors are due to the statically over-determined configuration [3]. In this paper it will be shown that only with certain restrictions can these correction formulas be applied to distributed loads.
The general construction of a force plate consists of a stiff plate resting on four posts (Fig. 1). In each of the four posts (i=1…4) there are tri-axial force transducers measuring compressive and tensile forces $F_{i,z}$ in the z direction, as well as shearing forces $F_{i,x}$ and $F_{i,y}$ in the x and y directions respectively. From these force signals the components of the resultant force $\underline{F} = [F_x, F_y, F_z]^T$ on the plate can be calculated by

$$F_x = \sum_{i=1}^{4} F_{i,x}, F_y = \sum_{i=1}^{4} F_{i,y}, F_z = \sum_{i=1}^{4} F_{i,z} \; . \qquad (1)$$

Because the deformations of the plate are small compared to the overall dimensions of the force plate, the z-coordinate of the point of force application is known to always be $z_{P,m} = z_P = h$. Therefore all further coordinates will only be given in the x and y directions. The coordinates $[x_{P,m}, y_{P,m}]^T$ of the point of force application calculated from the force signals are then given by the following two equations:

$$x_{P,m} = \frac{F_x \cdot h + a \cdot \left(F_{1,z} - F_{2,z} - F_{3,z} + F_{4,z}\right)}{F_z} \tag{2}$$

$$y_{P,m} = \frac{F_y \cdot h + b \cdot \left(F_{1,z} + F_{2,z} - F_{3,z} - F_{4,z}\right)}{F_z} \tag{3}$$

Further it is known, see also [3], that equations (2) and (3) are only valid for all points on the plate if the measurement posts are free of any moment in the plane of measurement.

METHODS: From experiments and theoretical considerations it is known that the systematic error

$$\Delta \underline{x}_P = \left[\Delta x_P, \Delta y_P\right]^T = \underline{x}_{P,m} - \underline{x}_P, \tag{4}$$

although computed from the force signals, is independent of the magnitude of the force for a single point loading [1]. Therefore the error function can be written in the form of nonlinear functions

$$\Delta \underline{x}_P = \underline{f}\left(\underline{x}_P\right) = \underline{g}\left(\underline{x}_{P,m}\right). \tag{5}$$

For the correction function $\underline{g}$ polynomial approximations of the form

$$\Delta x_P = \left(a_{1x} y_{P,m}^4 + a_{2x} y_{P,m}^2 + a_{3x}\right) \cdot x_{P,m}^3 + \left(a_{4x} y_{P,m}^4 + a_{5x} y_{P,m}^2 + a_{6x}\right) \cdot x_{P,m} \tag{6}$$

$$\Delta y_P = \left(a_{1y} x_{P,m}^4 + a_{2y} x_{P,m}^2 + a_{3y}\right) \cdot y_{P,m}^3 + \left(a_{4y} x_{P,m}^4 + a_{5y} x_{P,m}^2 + a_{6y}\right) \cdot y_{P,m} \tag{7}$$

are used [4]. A typical error pattern is given in Figure 2. There the measured data is interpolated using the above polynomials.

Because the plate and the measurement posts are very stiff, the deformations and strains are small. Therefore the problem can be described within the theory of linear elasticity. Furthermore, the signals from the force transducers are linear with respect to the magnitude of the measured forces.

A general solution can be found as a linear combination of single load cases. However, the COP depends only on the distribution $p(\underline{x})$ of the pressure normal to the surface of the plate. It is defined by

$$\underline{x}_{COP} = \frac{\iint_A p(\underline{x}) \cdot \underline{x} \cdot dA}{F_z} \quad \text{with} \quad F_z = \iint_A p(\underline{x}) \cdot dA. \tag{8}$$

For a given pressure distribution there is an infinitesimal force $dF_z = p\left(\underline{x}_P\right) \cdot dA$ at each point P of the plate, giving the infinitesimal contributions $dF_{i,x}$, $dF_{i,y}$ and $dF_{i,z}$ to the force signal. After some calculations it can be shown that the measured COP can be written as a function of the true COP using the error function for single point loading:

$$\underline{x}_{COP,m} = \frac{\iint_A p(\underline{x})\left(\underline{x} + \underline{f}(\underline{x})\right)dA}{\iint_A p(\underline{x})\,dA} = \underline{x}_{COP} + \frac{\iint_A p(\underline{x})\,\underline{f}(\underline{x})dA}{\iint_A p(\underline{x})dA}. \quad (9)$$

By correcting the measured COP with the correction function $\underline{g}$, the corrected value

$$\underline{x}_{COP,c} = \underline{x}_{COP,m} - \underline{g}\left(\underline{x}_{COP,m}\right) \quad (10)$$

in general differs from $\underline{x}_{COP}$ depending on the pressure distribution $p(\underline{x})$. The error of the corrected value can be calculated by

$$\Delta\underline{x}_{COP,c} = \underline{x}_{COP,c} - \underline{x}_{COP} = \frac{\iint_A p(\underline{x})\,\underline{f}(\underline{x})dA}{\iint_A p(\underline{x})dA} - \underline{g}\left(\underline{x}_{COP,m}\right). \quad (11)$$

From equation (11) it is directly evident that only when error functions $\underline{f}$ and $\underline{g}$ are linear is $\Delta\underline{x}_{COP,c}$ zero for all possible pressure distributions. That this is clearly not the case as given in [3] and [4].

RESULTS: For a KISTLER plate Type 9287B two load cases were investigated.
1. Two point loads of equal magnitude are separated by 160mm in the y-direction:
This load case was taken as a rough approximation of the load distribution in the mid-stance phase of gait. From the error plot (Fig. 3) it is clearly seen that the error in the measured COP is less than the error of a single point load (Fig. 2). If the correction formulas (7) and (8) are applied there will be a resultant error (Fig. 4). In general it can be said that the error will be overcompensated. Nevertheless the mean error after compensation (3.02±1.02mm) is less than for the uncompensated case (8.21±4.20mm).

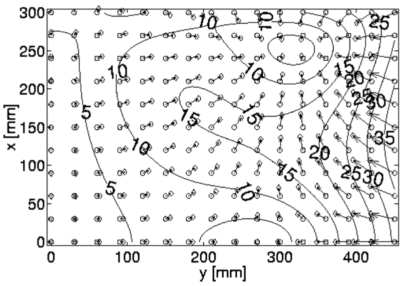

***Figure 2:** Error map for a KISTLER force plate Type 9287B with a single point load; (◦) true point of force application; (◊) "measured" point; the contour lines give the absolute value of the error in mm; because the error function is symmetrical with respect to the x and y axes, only the first*

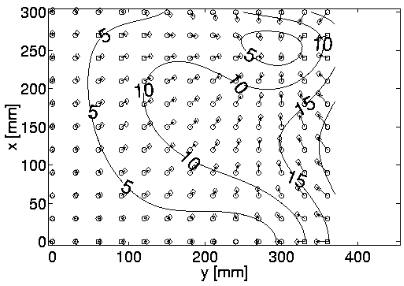

***Figure 3:** Error map for a KISTLER force plate Type 9287B with two point loads of equal magnitude and a separation of 160mm in the y-direction; (◦) true COP; (◊) measured COP.*

2. Uniform pressure over a x-y rectangle of 50x160mm:
In this case the errors after correction are less significant than in the first case (Fig. 5) when applying the correction formulas.

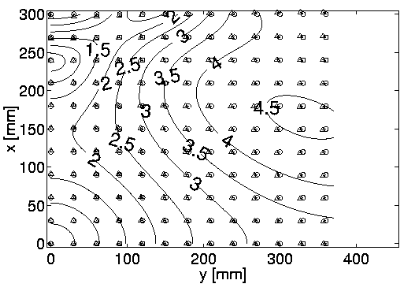

Figure 4: *Corresponding to Fig. 3 but after application of the correction formulas; (◦) true COP; (Δ) corrected COP.*

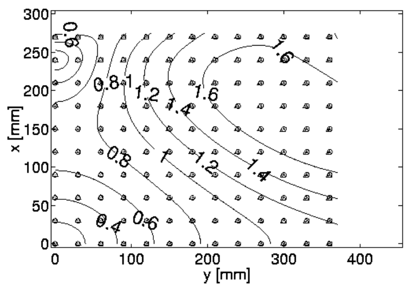

Figure 5: *Error map for a KISTLER force plate Type 9287B with uniform pressure over a rectangle of 50x160mm after application of the correction formulas; (◦) true COP; (Δ)*

DISCUSSION: The above results clearly show that when correcting systematic errors in the determination of the COP, one must be very careful about the distribution of the loads on the force plate. It can be concluded that accurate corrections can be made for forces evenly distributed over a small area. Errors are expected to be overcompensated much more, if there are only a few pressure peaks which are separated by a large distance. Especially when determining net joint torques in gait analysis one should keep in mind that these torques depend on accurate knowledge of the COP.
From a mechanical point of view these errors can be easily avoided by supporting the force plate in a statically determined way. However, such constructions lead to a less stiffer system and therefore to lower own frequencies.

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