QUALITATIVE AND QUANTITATIVE ANALYSIS OF MUSCLE POWER

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Muscle power is one mechanical quantity that is not clearly understood by many coaches and teachers of human movement. This is probably due to an inadequate differentiation between the concepts of work/energy and of power. Thus we will begin this paper with a discussion of these concepts. The analysis of the vertical jump as a measure of work, energy, and power will also be discussed.

THE CONCEPTS OF MECHANICAL WORK AND ENERGY

A motor task commonly used as a demonstration of muscle produced work and power is the vertical jump. There is a major problem, however, with this demonstration in that the height of a jump is produced by the instantaneous velocity of the jumper at the moment of release from the supporting surface and neither the height of the jump nor the release velocity are measures of power. These quantities are the products of the work that has been done during the act of jumping and not of power. In other words, the task of vertical jumping requires that a muscle produced force act upon the segments of the lower extremity through a distance which results in the production of mechanical energy. The amount of work done is equal to the amount of energy produced.

The analysis of a vertical jump requires that we measure three positions of the center of mass and two displacements. The three positions are \( Y_1 \), the lowest crouched position; \( Y_2 \), the extended position just as the body releases from the floor; and \( Y_3 \), the highest point of the jump. These positions are shown in Figure 1. The two displacements are defined from the position data as follows:

- The first displacement, which we will call height one \( (h_1) \) is the difference between positions one and two, i.e.,

\[
h_1 = Y_2 - Y_1
\]

(Equation 1).

This is the distance that the center of mass is raised during the support phase of the jump.
Figure 1. Three positions involved in the analysis of a vertical jump. These positions are $Y_1$, the lowest crouched position; $Y_2$, the extended position just as the body release from the floor; and $Y_3$, the highest point of the jump.

Figure 2. Two runners $A$ and $B$ obtain the same kinetic energy $E_k$, but runner $A$ does it in the shorter time $t_a$, while runner $B$ does it in longer time $t_b$. Runner $A$, therefore, is by definition the more powerful of the two.
The second displacement, which we will call height two \( h_2 \) is the difference between positions one and three, i.e.,

\[ h_2 = Y_3 - Y_1 \]  

(Equation 2).

This is the total distance that the center of mass is raised during the jump.

A third displacement, which is the height to which the center of mass is projected during the airborne phase of the jump, is defined as the difference between heights one and two, i.e.,

\[ \Delta h = h_2 - h_1 \]  

(Equation 3).

The Measurement of Work and Energy

The three displacements can be used in the measurement of work and the amount of energy produced during a vertical jump as follows:

--Work is defined as the product of force \( F \) and the distance \( d \) through which the force acts. Thus during a vertical jump the muscles of the lower extremities produce force through the first distance \( h_1 \) and thereby do work upon the jumper. In formula form this would be:

\[ \text{Work} = Fd = Fh_1 \]  

(Equation 4).

--Energy \( (E) \) is defined as the ability to do work. The two types of mechanical energy involved in a vertical jump are identified as gravitational potential energy \( (E_p) \) and kinetic energy \( (E_k) \). These two forms of energy are defined as follows:

--Gravitational Potential Energy

\[ E_p = mgh_2 = mgh_1 + mg\Delta h \]  

(Equation 5).

When the center of gravity of the jumper is raised to \( h_2 \), the force of gravity \( (mg) \) then does work through that distance in bringing the body of the jumper back to the starting point \( (Y_1) \). Thus the muscles do work in raising the body to \( h_1 \), and gravity does work in lowering the body back through the distance \( h_2 \).
--Kinetic Energy

\[ E_k = \frac{1}{2}mv^2 \]  
(Equation 6).

When the body of the jumper releases from the surface it has the velocity of release \( V_r \), which gives the body its kinetic energy \( \frac{1}{2}mv_r^2 \) and it is this energy that then does the work of projecting the body into the air to the height \( \Delta h \).

The kinetic energy finally ends up as potential energy \( (mg\Delta h) \) at the top of the jump. It should be remembered that the potential energy at the top of the jump \( (mg\Delta h) \) is equal in magnitude to the kinetic energy at release \( (\frac{1}{2}mV_r^2) \). The total amount of work that the muscles produce can be measured through Newton's Second Law expressed in terms of work/energy relationships as:

\[ \text{Work} = \text{Fd} = Ph_1 = mgh_1 + \frac{1}{2}mv_r^2 \]  
(equation 7).

This equation shows that the work done during a vertical jump when muscle forces act through a distance \( Fh_1 \) is equal to the gravitational potential energy plus the kinetic energy that it produces.

**THE CONCEPT OF POWER AND ITS SIGNIFICANCE**

The measurement of power requires us to divide both sides of equation 7 by the time \( t \) to obtain

\[ \text{Power} = \frac{Ph_1}{t} = \frac{mgh_1}{t} + \frac{1}{2}mv_r^2 \]  
(equation 8).

The right side of equation 8 is especially interesting because it gives the rate of energy production. In many human performance situations it is the rate of kinetic energy production that is the most important.

A given amount of kinetic energy can be produced by either a large force acting through a short distance or a smaller force acting through a longer distance. Applying the force, however, through the longer distance will usually require more time so that the rate at which the kinetic energy is produced is less and by definition there is less muscle power.

We can use sprinting as an example of a power event. Let us assume that we have two sprinters (A and B) and that each has the same mass and each is capable of producing the same kinetic energy. If we construct a kinetic energy/time graph it might look like that shown in Figure 2. Both runners A and B develop the same kinetic energy, but runner A does it more rapidly and is, therefore, the more powerful of the two.

When execution time is a limiting factor in performance as in baseball batting and in sprinting, how fast one can develop kinetic energy of either the body as a whole or of its parts becomes a crucial factor. Thus because of the importance of power in various types of performance it is essential that we have a valid method of measuring it.
ANALYSIS OF THE VERTICAL JUMP

Two papers were presented at the third ISBS Symposium held in Greeley, Colorado last year (1985) that describe some of our work at the University of Northern Colorado in the analysis of muscle power. The first was a paper by Paul A. Lightsey, Department of Physics, University of Northern Colorado in which he presented an original formula for the measurement of power from displacement data obtained from a filmed vertical jump.

The second paper was by Shetty, et. al., (1985) which described a pilot study designed to validate the Lightsey formula. A third paper in this series, which summarizes the first two papers and reports the results of a followup study is presented by Shetty and Barham (1986) elsewhere in these proceedings. In this third paper it is concluded that the displacement based algorithms developed by Lightsey is valid and that it can be used as a reliable and inexpensive method of measuring muscle power.

The Lightsey algorithms can also be used to measure average force, average velocity, duration of the support phase of the jump, as well as the average work/energy and power involved in the jump.

The Analysis of Average Force

--Proposition: work done by the muscles of the lower extremity during the support phase of the jump is equal to the potential energy of the center of mass at its greatest height in the air, i.e.,

\[ \bar{F} \cdot h_1 = mgh_2 \]

so average force \( \bar{F} \) is

\[ \bar{F} = \frac{mgh_2}{h_1} \]  (Equation 9).

--The ratio \( h_2/h_1 \) can be put in a different form as follows:

--According to the principles of parabolic motion, the vertical displacement (\( \Delta h \)) of the jumper after release from the surface is given by:

\[ \Delta h = \frac{v_r^2}{2g} \]  (Equation 10).

--The height (\( h_1 \)) that the center of mass is raised during surface contact is given by:

\[ h_1 = \frac{v_r t}{2} \]  (Equation 11).

--So:

\[ \frac{h_2}{h_1} = 1 + \Delta h = 1 + \frac{v_r^2}{2g} \]  (Equation 12).
When Equation 12 is substituted into Equation 9 we obtain
\[ F = mg(h_2/h_1) = mg(l+a/g) = mg + ma \]  
(Equation 13).

This equation (13) shows that the total average force produced by the muscles of the lower extremity during the support phase of the jump is equal to the force required to move the body weight (mg) plus the net force that accelerates the body mass (ma) to the velocity at which the jumper releases from the surface \( V_r \). It is the release velocity \( V_r \), of course, that produces the height of the jump (eq. Equation 10).

Analysis of Average Velocity

--Average velocity \( \bar{V} \), when the original velocity is zero, is given by
\[ \bar{V} = \frac{V_r + V_o - 1/2V_r}{2} \]  
(Equation 14).

--Rearranging Equation 10 we obtain the release velocity \( V_r \) as being equal to
\[ V_r = \sqrt{2ah_1} \]  
(Equation 15).

--The velocity of release is also given by
\[ V_r = \sqrt{2ah_1} \]  
(Equation 16).

where \( a \) is net acceleration, i.e., \( a = F/m - g \)  
(Equation 17).

--Substituting Equation 17 into Equation 16 we obtain
\[ V_r = \sqrt{2(F/m - g)h_1} \]  
(Equation 18).

--Substituting Equation 18 into Equation 14 we obtain
\[ \bar{V} = 1/2 \sqrt{2(F/m - g)h_1} = \sqrt{1/2(F/m - g)h_1} \]  
(Equation 19).

--Substituting Equation 9, from the section on force analysis, into Equation 19 we obtain
\[ \bar{V} = \sqrt{1/2 \left[ mg(h_2/h_1) - g \right] h_1} \]

Thus the two variables that determine the average velocity of the jumper during the support phase of the jump are the magnitudes of \( h_1 \) and \( h_2 \).
Analysis of the Duration of the Support Phase

--The duration \( t \) of the support phase of the jump is equal to \( h_1 \) divided by the average velocity \( \bar{V} \), i.e.

\[
\frac{h_1}{\bar{V}} \quad \text{(Equation 21)}.
\]

--Substituting Equation 20 into Equation 21 we obtain

\[
t = \frac{h_1}{\bar{V}} = \frac{h_1}{\sqrt[1/2]{2g\left(h_2/h_1-1\right)h_1}} \quad \text{(Equation 22)}.
\]

Analysis of Work and Power

--Power has been defined as the rate of doing work and according to Equation 8 it is given by

\[
\bar{P} = \frac{F \cdot h_1}{t}
\]

--When the average force equation (#9) and the time equation (#22) are substituted into Equation 8 we obtain:

\[
\bar{P} = \frac{F \cdot h_1}{t} = \frac{mg(h_2/h_1)^{1/2}h_1}{h_1/\sqrt[1/2]{2g\left(h_2/h_1-1\right)h_1}} = \frac{mg\sqrt{2g\left(h_2/h_1-1\right)h_1}}{h_1} = \frac{mg(h_2/h_1)^{1/2}h_1}{\sqrt[1/2]{2g\left(h_2 - h_1\right)}} = \frac{mg(h_2/h_1)^{1/2}h_1}{\sqrt[1/2]{2g\left(h_2 - h_1\right)}} \quad \text{(Equation 23)}.
\]

Equation 23 is Lightsey’s displacement based algorithm for the calculation of muscle power from the two height measures obtained during a vertical jump.
