THE IDENTIFICATION OF IMPULSES IN 3-D RECONSTRUCTED DATA USING RECURSIVE FILTERS ON THE ORIGINALLY DIGITIZED 2-D IMAGE DATA

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INTRODUCTION: It is usual to reconstruct three-dimensional (3-D) data from two or more sets of two-dimensional (2-D) data before smoothing it. Unfortunately most smoothing techniques are essentially low-pass filters (Butterworth, quintic spline etc.) and thus remove all the high frequency components. Although the results may be aesthetically pleasing the filters provide no implicit indication that the smoothed data is more accurate than the original data and they also remove any discontinuities (Figure 1) along with the noise. A discontinuity would result from the application of an impulse to a point and is related to fundamental actions such as foot strike. Given a typical video sampling frequency of 50 Hz there is also a very high probability that the impulse would not be explicitly recorded (Figure 2), requiring additional computation to estimate when and where it did actually occur.

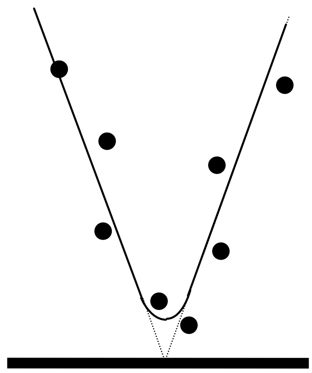

Figure 1. Low-pass Filter, discontinuity is smoothed out.

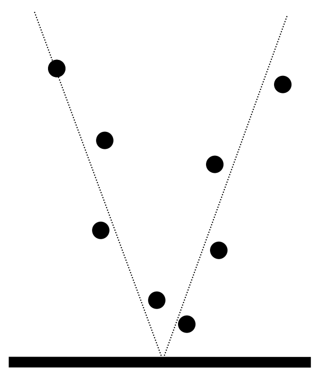

Figure 2. Impulse not explicitly recorded.

To reconstruct 3-D data one usually has more equations than unknowns - the system is over-determined and can be solved by least squares estimation. In a perfect reconstruction the condition of coplanarity would be satisfied and there would be no residual error. Smoothing the data prior to reconstruction should improve its quality and this would be confirmed by a lower residual error. An increasing residual error would imply that the 2-D data are diverging and the smoothing is making the data unreliable. Processing the data in both a forward and backward direction enables the discontinuity to be detected and extrapolating the two resultant data sets until they intersect should be a good estimate of the original point of discontinuity. If the same discontinuity could be located independently on more than one set of 2-D data then the discontinuities could be used to measure the phase difference between the cameras. This bidirectional filtering of the data can be achieved by a recursive filter.

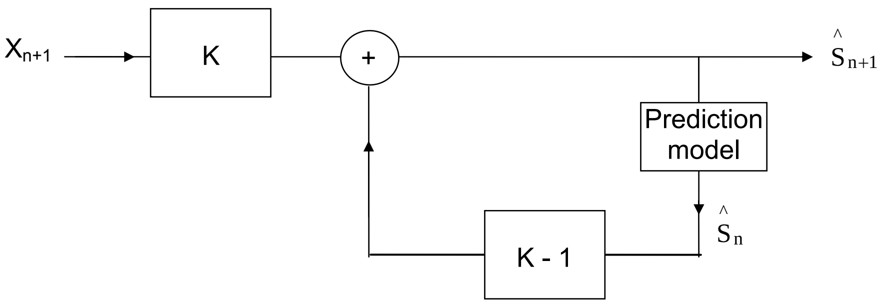

Figure 3. Block diagram of a simple Kalman Filter

METHOD: A Kalman filter estimates the value of a variable, $\hat{S}_{n+1}$, at time (n+1) by combining the value predicted from a previous estimate $\hat{S}_n$, with the current measured value, X_{n+1} :

$\hat{S}_{n+1} = (1 - K)\hat{S}_n + K X_{n+1}$

where K is a constant. If the previous estimates were considered good then $\hat{S}_n$ would be the favoured term, making K closer to zero. Conversely, if previous predictions were poor then more notice would be taken of the measured value X_{n+1} and K would approach unity. Assuming the signal to be in a steady state prior to the discontinuity, the value of K would converge to a constant value. At a discontinuity the predicted value would diverge from the measured value (Figure 4), causing K to increase. K will only decrease when the signal has returned to a state condition. The variation of K is dependent on both the noise of the measurement and the extent of the discontinuity. If the filter were applied in the reverse direction the same effect would happen, but in the opposite direction, thus confirming the existence of a discontinuity.

Having located the discontinuity, the original data can be treated as two separate sets of data which are then extrapolated using their respective prediction models to find their point of intersection. Prediction of the state $\hat{S}_{n+1}$ from the state $\hat{S}_n$ was done from the equation

$$s = ut + 0.5at^2$$

leading to the difference equation

$$\hat{S}_{n+1} = (1 - K)(\hat{S}_n + \hat{V}_n \Delta t + 0.5 \hat{A}_n \Delta t^2)$$

where $\hat{V}_n$ and $\hat{A}_n$ are estimates of the velocity and acceleration respectively. K is calculated from the equation

$$K_{n+1} = \frac{\varepsilon(n+1)}{\sigma_w^2}$$

where $\varepsilon(n+1)$ is the calculated mean square error between the original noise free signal, S_n and its estimate $\hat{S}_n$, while σ_w^2 is the variance of the measured signal noise.

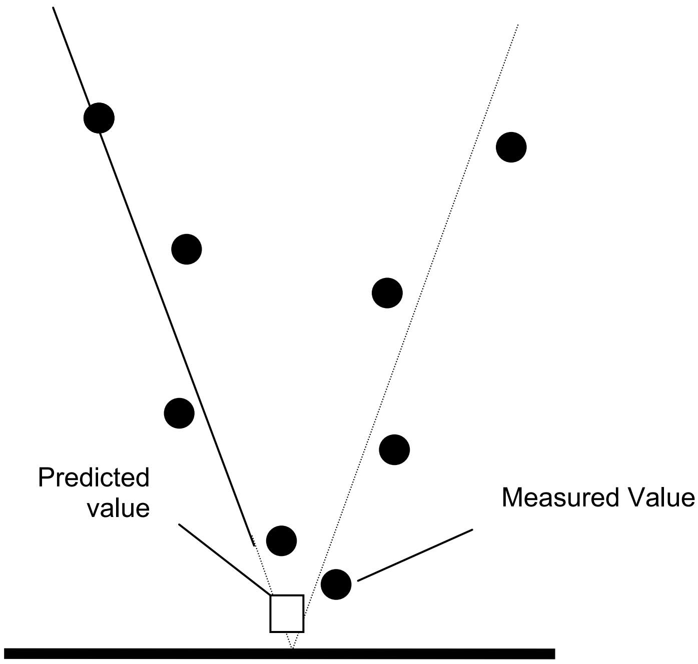

Figure 4. Divergence of the predicted and measured values.

PROCEDURE: Two 50Hz video cameras were located with their optical axes approximately orthogonal to each other and a 3-D volume was calibrated using a Peak Performance calibration frame. A table was then placed in the calibration volume which was used to bounce golf balls off. The slave camera was connected to the master camera and set up to film with phase differences of 0°, 90°, 180° and 270° with respect to the master camera. Using a GE Imager LCD10E video projector the image co-ordinates were generated by a Quora digitiser. The 3-D data was reconstructed using an 11 parameter Direct Linear Transform (Abdel-Aziz & Karara, 1971) and the residual error calculated from the pseudo-inverse data matrix.

RESULTS: The calculated time between bounces for any two camera combinations was accurate to within 0.01 seconds. The phase of the cameras did not appear to have any effect.
The residual error of the reconstructed showed an improvement of between 2% and 8%.

DISCUSSION: As the time between impulses was consistently measured by all the cameras it would appear that the phase difference between any two asynchronous cameras can be calculated - provided they both record the same impulse. The

accuracy of this calculation is highly dependent, though, on the quality of the prediction model which is also required to estimate the best-fit location of the phase-shifted markers to enable 3-D reconstruction of the asynchronous markers.
Although the residual error was reduced in all cases after filtering, a larger decrease was anticipated. Having calculated when and where the discontinuities occurred it should be possible to apply a low pass filter to the intermediate points. This approach is made difficult by the constraint of having to ensure that the new curves still intersect at the same points of discontinuity, whilst providing a best fit to the other points.

CONCLUSIONS: This technique worked very well when identifying obvious points of discontinuity such as occurred with the bouncing ball. It remains to be seen how easy it will be to determine lesser discontinuities such as those that occur at foot strike. Using the information gained is surprisingly hard to utilise with the ability to measure the phase difference between cameras being limited by the quality of the prediction models operating on a distorted image.
The small reduction in the residual error may be an indication that smoothing the 2-D data prior to reconstruction is not effective, though there is the continued problem of determining the accuracy of data smoothed after reconstruction. Alternatively, the comparatively small reduction in residual error may be an indication of how large the systematic errors are. These errors will continue to dominate the quality of the reconstructed data until they can be effectively compensated for.

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